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The effect of reradiation on heat transmission

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THE EFFECT OF RERADIATION

ON

HEAT TRANSMISSION

BY

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A

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NOMENCLATURE

A	Surface area	Sq.ft.
a	Distance	ft.
D	Diameter	ft.
P	Perimeter of the enclosure wall	ft.
q	Quantity of heat	Btu/hr.
S_1	Radiator or source	
S_2	Sink	
T	Temperature	^oR
W	Radiant heat flux or emissive power	Btu/(hr.)(sq.ft.)
x	Distance of any element	ft.
X	Ratio of distance to diameter	dimensionless
β	Angle	radians
σ	Boltzmann constant - 0.1713×10^{-8}	Btu/(hr.)(sq.ft.) ($^oR^4$)
f(x)	Factor for radiation from one surface to another surface element of the refractory wall	
F(x)	View factor, shape factor, or angle factor	
Ø(x)	Factor for direct radiation from source to surface element of the enclosing wall	

Indices

a	Small strip
b	Black body
s	Surface
R	Reradiating wall

I. INTRODUCTION

In heat transmission by radiation, it is very easy to find the heat transmitted from one surface to another. Much work has been done in this area. The problem becomes complicated when the two surfaces are enclosed or joined by an enclosure. Such problems frequently arise in the design of furnaces. One problem is linked with the economy of fuel, by proper consideration of heat losses through the openings in furnace walls such as doors and burner openings. A second problem is associated with the heating capacity and useful heat transfer through the openings such as heat ports in walls or perforations in arches or muffles. The performance of refractory walls is of interest here.

In heat transmission through surfaces enclosed by refractory enclosures, the problem involves not only direct radiation between source and sink but also the heat reradiated from the enclosing walls. The amount of reradiation depends upon the geometry of all the surfaces involved. The complication of the problem still increases due to the fact that the temperature of the refractory enclosure varies from point to point. The usual method is to assume a temperature for the reradiating walls comparable with experience. This could be the mean temperature of the source and sink or some other approximate method.

The purpose of this paper is to illustrate a method to account for reradiation by enclosure walls. A special case of cylindrical enclosure with disc type source and sink at the ends is considered here. The same method can be applied to enclosures of different shapes with appropriate modifications in dimensions and view factors. Specifically it is desired to determine the temperatures of the refractory surface at all points and compare this temperature with some of the usual assumptions.

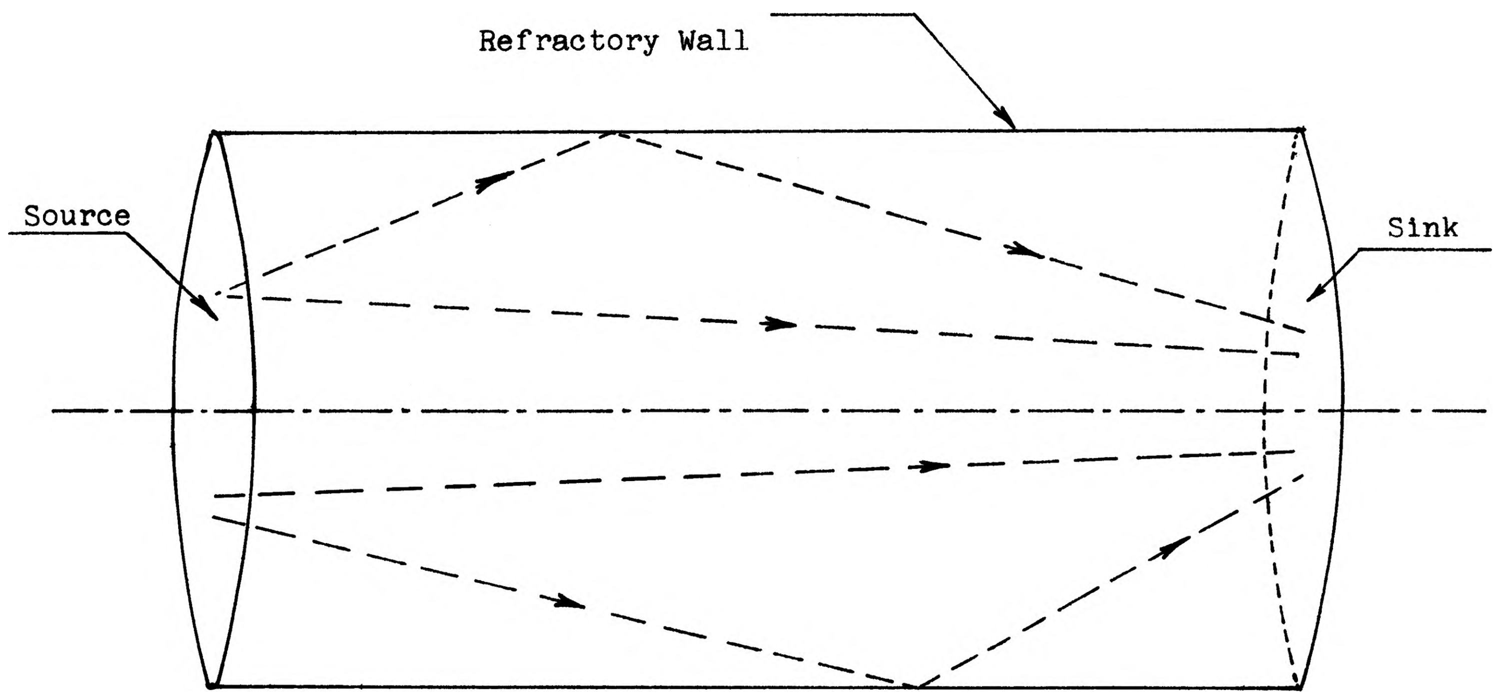

Fig. 1 Heat Transmission From One Surface to Another
Both Enclosed by Refractory Wall

II. REVIEW OF LITERATURE

Various schemes have been used to handle this problem. In many books[6][7], the problem is fairly simplified by considering the refractory surface of the enclosing walls at uniform temperature and thus assume some approximate value for the temperature. The word refractory is used to indicate that the net radiant heat transfer of the wall surface is zero.

The variation in temperature of refractory walls was known as early as in 1923 by W. Trinks[8] in his book of "Industrial Furnaces". He reasoned that it must be possible to find a position for the radiating surface such that the black body radiation of the source multiplied by $(\frac{i}{180})^2$ is equal to the actual radiation, where i is the angle subtended by the outer edges at that position, (See fig. 2). He again found that the position of the above mentioned plane is at a length of 0.45 times wall thickness from the inner edge. The factor $(\frac{i}{180})^2$ can be replaced by $(\frac{\text{Vertical angle}}{180} \times \frac{\text{Horizontal angle}}{180})$. Thus by knowing the vertical angle and the horizontal angle subtended at the center of the plane located at the distance of 0.45 X thickness of the wall from the inner edge, the actual radiation can be calculated.

A second attempt was done by J. D. Keller[5] in his paper published in "Fuels and Furnaces" - December 1927. He

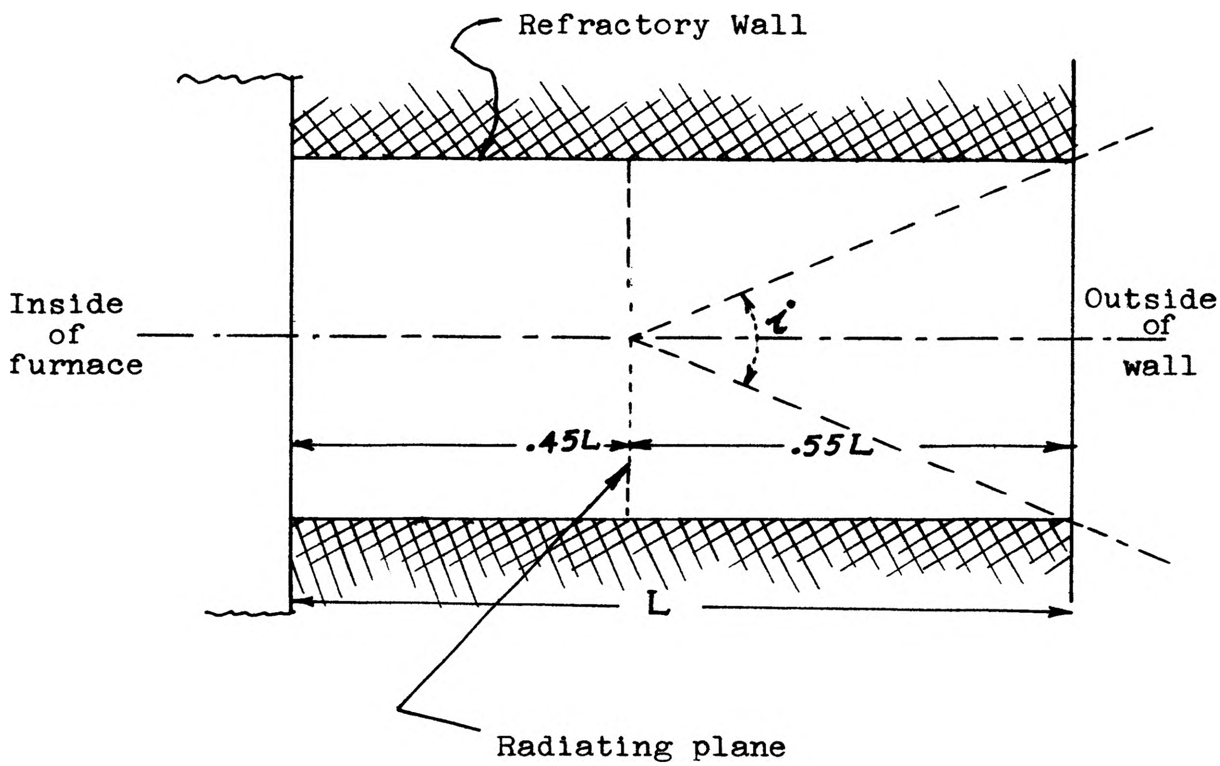

Fig. 2 Location of Equivalent Radiating Plane

obtained the factors which express the reradiation as a fraction of that from a fully exposed surface having various ratios of diameter to the wall length.

The results obtained by Trinks are fairly accurate for large included angles, that is where the wall length of the enclosure is very small compared to the height and width. But the values are rather low for long enclosures or thick walls.

The factors obtained by Keller are fairly accurate for round and square openings but it fails to account for rectangular and other shapes of openings.

Another remarkably successful attempt to account for reradiation has been done by H. C. Hottel and J. D. Keller[3] in their paper published in A.S.M.E. transactions - 1933. They developed accurate methods of calculation and presented the result in form of curves from which the reradiation and total radiation factors can readily be known for various standard shapes of openings and enclosures.

Hottel and Keller assumed the linear variation of the radiant heat flux or emissive power in their paper. It is shown in this thesis that the variation in radiant heat flux is not exactly linear. One of the two methods suggested by Hottel and Keller is used to solve the integral equations.

III. DISCUSSION

1. Assumptions.

Before proceeding to solve the actual problem, it is necessary to make some assumptions which do not affect the accuracy.

(A) Source S_1 and sink S_2 are perfectly black bodies.

(B) Condition of steady state has been reached. This means that the temperatures of the source, sink and every point of the enclosure are steady.

(C) The enclosing wall is such that the conduction through the walls and between S_1 and S_2 can be neglected compared to the large radiant heat transfer.

(D) Enclosure of the refractory wall is assumed to be a perfect reradiating surface. In other words there is no net flux and all the radiant flux absorbed by the wall is emitted back to the interior space.

2. Theory.

If W_{b1} and W_{b2} are radiant heat flux or emissive powers of source S_1 and sink S_2, then by Stefan-Boltzman's law

$$W_{b1} = \sigma T_{s1}^4 \text{ and } W_{b2} = \sigma T_{s2}^4 \qquad (1)$$

where σ = Boltzman's constant = 0.1713×10^{-8} Btu/ft^2.hr.°R^4

Assuming there is no enclosure wall, the heat exchanged between S_1 and S_2 can be given by

$$q = (W_{b1} - W_{b2})\ A_1\ F_{1-2} \qquad (2)$$

where F_{1-2} is defined as the shape factor, view factor or angle factor and is equal to the fraction of radiation emitted by S_1 and received by S_2 depending upon the area of the two surfaces and the solid angle filled by radiant rays.

It has been found that $F_{1-2} = \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi S^2} dA_2$ (3)

where S is the distance between two small surface elements of the radiating bodies and the angles between the two normals and connecting line are β_1 and β_2 (as shown in Fig. 3).

From the above fundamental equation, shape factors for two equal and parallel circular discs at a distance L is given by $F_{1-2} = 1 + 2X^2 - 2X(X^2 + 1)^{\frac{1}{2}}$ (4)

where $X = \frac{\text{Length}}{\text{Diameter}} = \frac{L}{D}$

In the presence of enclosure walls, part of the heat radiated from source S_1 is absorbed by refractory wall and part of it will be absorbed by S_2. The heat absorbed by refractory wall will be reradiated.

Consider a small element of area δA of the refractory wall at distance 'a' from the inner edge. (See Fig. 4).

The total heat radiated to area δA is equal to the heat radiated and reflected from δA.

If $\phi(x)$ is a function of the direct radiation from the surface S_1 to the surface element of area δA at a distance x, the heat radiated from source S_1 to δA is given by $W_{b1}\phi(a)\delta A$.

Similarly heat radiated from the surface S_2 to the element δA is $W_{b2}\phi(L-a)\delta A$.

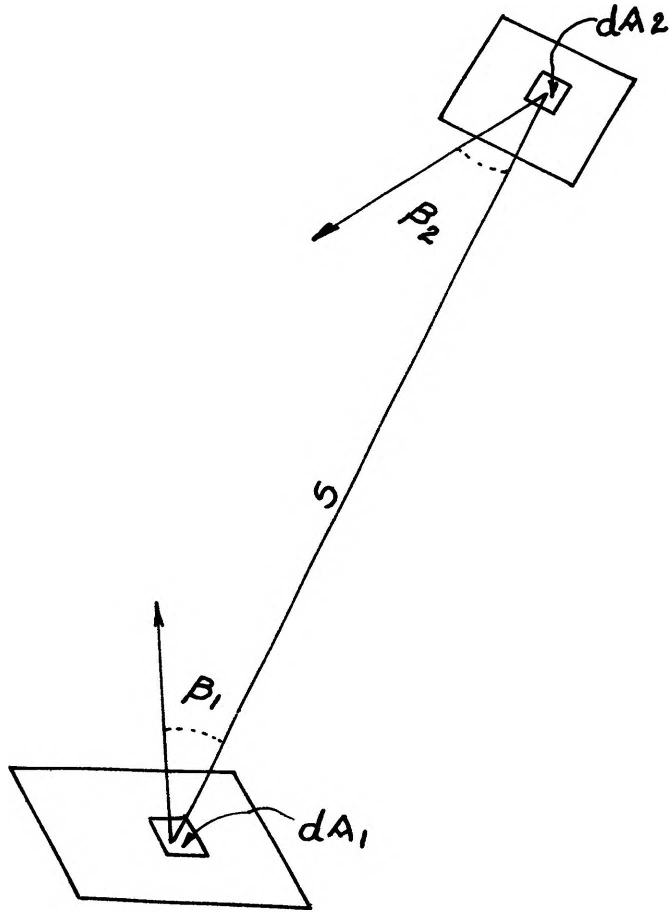

Fig. 3 Heat Exchange by Radiation Between Two Surface Elements

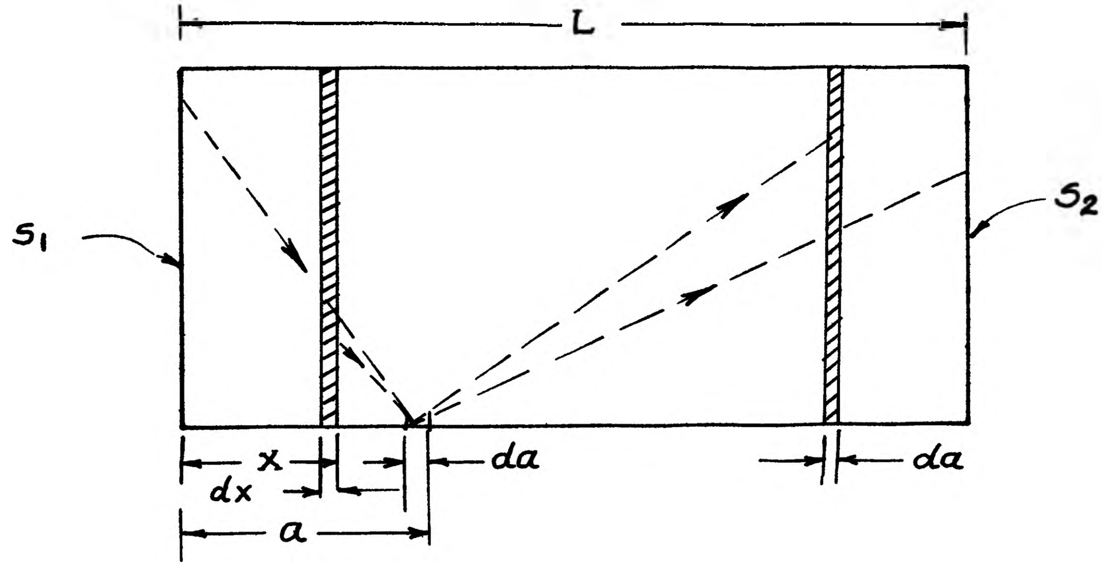

Fig. 4 Reradiation From Refractory Walls

Let $f(x)\,\delta x'$ be the fraction of radiation from an element δA which is intercepted by another element of area $\delta A'$ of length $\delta x'$ and separated from the first by normal distance $(a-x)$. Therefore radiation to δA from the sides between $x=0$ to $x=a$ is $\int_0^a W_x\, f(a-x)\,\delta A\, dx$.

Similarly the heat radiated to δA from the sides between $x=a$ to $x=L$ is $\int_a^L W_x\, f(x-a)\,\delta A\, dx$.

Now the heat radiated from element δA in all the directions is $W_a . \delta A$.

Therefore from the heat balance we can write an equation

$$W_a\,\delta A = W_{b_1}\phi(a)\,\delta A + \int_0^a W_x f(a-x)\,\delta A\, dx + \int_a^L W_x f(x-a)\,\delta A\, dx + W_{b_2}\phi(L-a)\,\delta A \quad (5)$$

Dividing the equation (5) through out by δA,

$$W_a = W_{b_1}\phi(a) + \int_0^a W_x f(a-x)\,dx + \int_a^L W_x f(x-a)\,dx + W_{b_2}\phi(L-a) \quad (6)$$

But heat radiated from S_1 to δA is given by $W_{b1}\ A\ F_{1-(\delta a)}$

$$W_{b1}\ A\ F_{1-(\delta a)} = W_{b1}\ \phi(a)\delta A \quad \text{(By principle of reciprocity)}$$

$$A\ F_{1-(\delta a)} = \phi(a).\delta A$$

$$A\ F_{1-(\delta a)} = \phi(a)P.\delta a$$

where P is the perimeter.

Thus there exists a definite relationship between the function $\phi(x)$ and the fraction $F(x)$.

If the fraction $F(x)$ of radiation from S_1 is intercepted by S_2, the fraction $-\frac{dF(x)}{dx}\,\delta x$ will be intercepted by a small strip on the side-wall of width δx, lying between the two plane areas S_2 located at distance x and x- δx respectively from radiator S_1. The negative sign indicates that F(x) decreases with increase in x.

By the principle of reciprocity,

$$A(-\frac{dF(x)}{dx}\delta x) = P.\delta x\ \phi(a)$$

$$\phi(x) = -\frac{A}{P}\cdot\frac{dF(x)}{dx} = -\frac{A}{P}F'(x) \qquad (7)$$

Similarly, if $\phi(x)$ represents the fraction of the radiation from δA, intercepted by A at a distance x from δA, $-\frac{d\phi(x)}{dx}\cdot\delta x'$ represents the fraction intercepted by the strip of area $\delta A' = P.\delta x'$, between two surfaces S_2 located at distances x and x- δx from radiator δA. This fraction by definition is equal to $f(x)\delta x'$.

$$f(x)\delta x' = -\frac{d\phi(x)}{dx}\delta x'$$

$$f(x) = -\frac{d\phi(x)}{dx} = +\frac{A}{P}F''(x) \qquad (8)$$

Then the net exchange of heat transfer between surfaces S_1 and S_2 can be determined from the following equation.

$$q_{net} = (W_{b1} - W_{b2})\ AF_{1-2} + \int_0^L (W_{b1}-W_a)\ \phi(a)\ P.\delta a \qquad (9)$$

Now knowing W_a from equation (6), net heat transfer between S_1 and S_2 can be calculated.

To calculate W_a, the wall surface is assumed to be divided into a number of small strips and the temperature in each strip is assumed to be constant.

Again from equation (4), for equal and parallel discs

$$F(x) = 1 + 2X^2 - 2X(X^2+1)^{\frac{1}{2}}$$

where $X = \frac{x}{d}$.

$$\phi(x) = -\frac{A}{P} F'(x)$$

For a circular disc the area $A = \frac{\pi D^2}{4}$ and the perimeter $P = \pi D$

$$\frac{A}{P} = \frac{D}{4}$$

$$\phi(x) = -\frac{D}{4} F'(x)$$

$$= \frac{(X^2 + \frac{1}{2})}{(X^2 + 1)^{\frac{1}{2}}} - X \qquad (10)$$

Similarly, $f(x) = -\phi'(x)$

$$= \frac{1}{D} \left(1 - \frac{X(X^2 + 3/2)}{(X^2 + 1)^{3/2}}\right) \qquad (11)$$

Substituting the values from equations (1), (10) and (11) in the equation (6), the radiant heat flux W_a can be found. But to solve the integral equation the wall surface is assumed to be divided into a number of strips and the temperature for any particular strip is assumed to be constant. Thus radiant heat flux for each strip will remain constant at any point on that strip. By taking more and more strips the correct temperature can be found. This method is illustrated in the following example.

3. Example.

$T_{s1} = 3000^{\circ}R$, $L = 10$ ft.

$T_{s2} = 700^{\circ}R$, $D = 5$ ft.

$\frac{L}{D} = 2.$

Assume the wall surface to be divided into ten strips as shown in Figure 5. Each strip is assumed to be at a constant temperature.

$$W_{b1} = \sigma T_{S1}^{4}$$
$$= 0.1713 \times 10^{-8} (3000)^4$$
$$= 138{,}510 \text{ Btu/hr.ft}^2 \qquad (12)$$
$$W_{b2} = \sigma T_{s2}^{4}$$
$$= 0.1713 \times 10^{-8} (700)^4$$
$$= 410.571 \text{ Btu/hr.ft}^2 \qquad (13)$$

From equation (11),

$$f(a-x) = \frac{1}{D} \left(1 - \frac{(a'-X)\ (a'-X)^2 + 3/2}{\left[(a'-X)^2 + 1 \right]^{3/2}} \right)$$

where $a' = \frac{a}{D}$

$$\int_0^a f(a-x)dx = \left[X + \frac{2(a'-X)^2 + 1}{2\left[(a'-X)^2 + 1\right]^{\frac{1}{2}}} \right]_0^{a'} \qquad (14)$$

Similarly,

$$\int_a^L f(x-a)dx = \left[X - \frac{2(X-a')^2 + 1}{2\left[(X-a')^2 + 1\right]^{\frac{1}{2}}} \right]_{a'}^{L} \qquad (15)$$

From equation (10),

$$\phi(a) = \frac{(a'^2 + \frac{1}{2})}{\left[a'^2 + 1\right]^{\frac{1}{2}}} - a' \qquad (16)$$

and

$$\phi(L-a) = \frac{(L'-a')^2 + \frac{1}{2}}{\left[(L'-a')^2 + 1\right]^{\frac{1}{2}}} - (L'-a') \qquad (17)$$

where $L' = \frac{L}{D}$

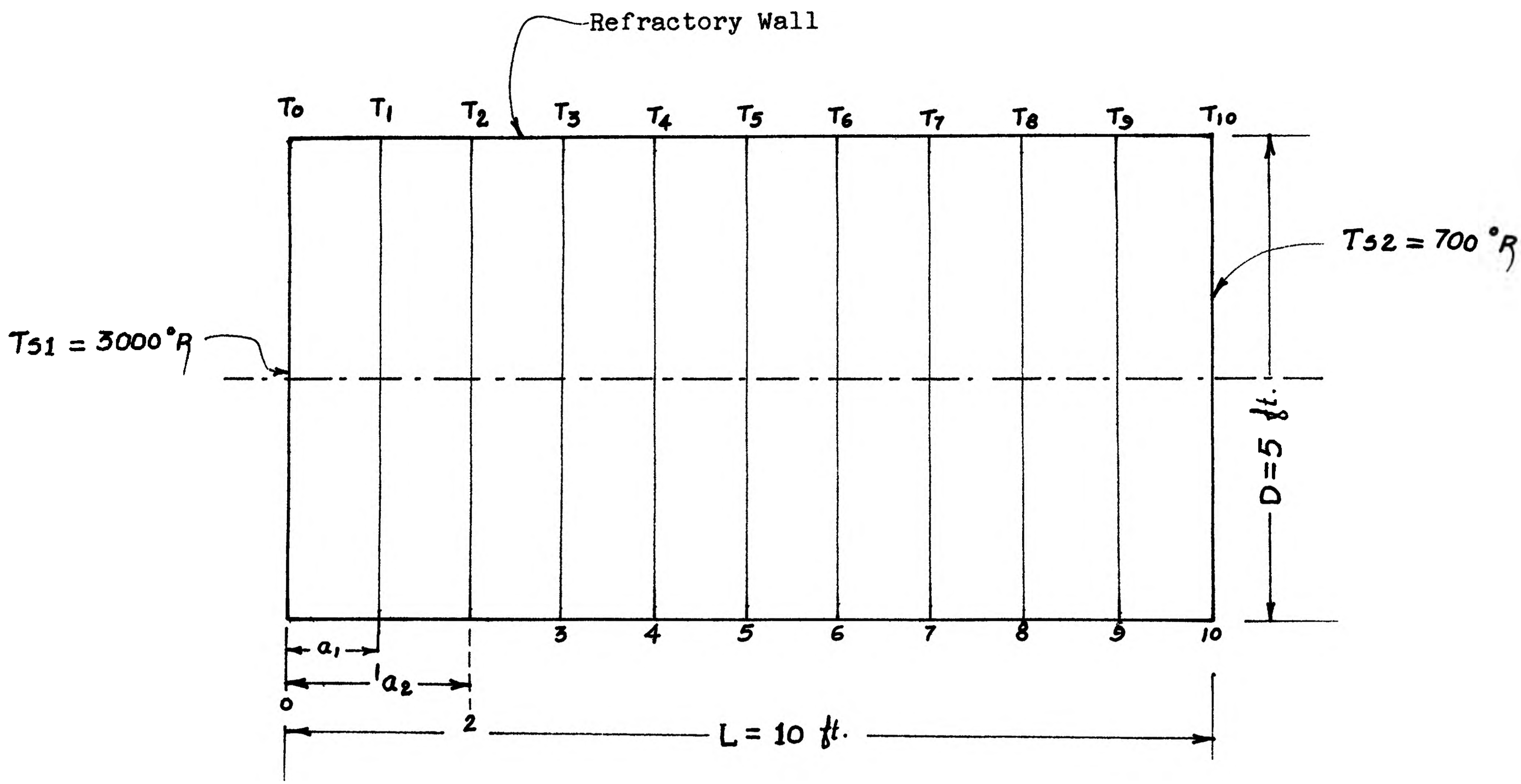

Fig. 5 Nomenclature for Cylindrical Enclosure

Now for the first strip, at point 1 the radiant heat flux is W_{a1} and from equation (6),

$$W_{a_1} = W_{b_1}\,\phi(a_1) + \int_0^{a_1} W_x\, f(a_1 - x)\,dx + \int_{a_1}^{L} W_x\, f(x - a_1)\,dx + W_{b_2}\,\phi(L - a_1)$$

$$\begin{aligned} W_{a_1} = {} & W_{b_1}\,\phi(a_1) + W_{a_1}\int_0^{a_1} f(a_1 - x)\,dx + W_{a_2}\int_{a_1}^{a_2} f(x - a_1)\,dx \\ & + W_{a_3}\int_{a_2}^{a_3} f(x - a_1)\,dx + W_{a_4}\int_{a_3}^{a_4} f(x - a_1)\,dx + W_{a_5}\int_{a_4}^{a_5} f(x - a_1)\,dx \\ & + W_{a_6}\int_{a_5}^{a_6} f(x - a_1)\,dx + W_{a_7}\int_{a_6}^{a_7} f(x - a_1)\,dx + W_{a_8}\int_{a_7}^{a_8} f(x - a_1)\,dx \\ & + W_{a_9}\int_{a_8}^{a_9} f(x - a_1)\,dx + W_{a_{10}}\int_{a_9}^{a_{10}} f(x - a_1)\,dx + W_{b_2}\,\phi(L - a_1) \end{aligned} \quad \text{—(18)}$$

Similarly nine other equations were obtained for W_{a2}, W_{a3}, W_{a4}, W_{a5}, W_{a6}, W_{a7}, W_{a8}, W_{a9}, and W_{a10}. By substituting the values from equations (12),(13),(14),(15),(16) and (17) in the equations for W_{a1}, W_{a2}, ...W_{a10} the ten equations were obtained where $a_1 = \frac{a}{D} = \frac{1}{5} = 0.2$, $a_2 = \frac{2}{5} = 0.4$, $a_3 = 0.6$, $a_4 = 0.8$, $a_5 = 1.0$, $a_6 = 1.2$, $a_7 = 1.4$, $a_8 = 1.6$, $a_9 = 1.8$ and $a_{10} = 2.0$. Solving ten equations simultaneously, the values of W_{a1}, W_{a2}, W_{a3}, W_{a4}, W_{a5}, W_{a6}, W_{a7}, W_{a8}, W_{a9}, W_{a10} were obtained. The computer program was set up for these equations. (For the computer program, see Appendix)

Also $W_a = \sigma T^4$

Therefore knowing the values of W_{a1}, W_{a2}, ...W_{a10} the temperatures T_1, T_2, T_3, T_4, ...T_{10} can be calculated.

No.	$\frac{a}{D}$	W_a Btu/hr.ft^2	T ^oR
1	0.2	92,630	2711.75
2	0.4	80,531	2618.49
3	0.6	69,519	2523.99
4	0.8	59,549	2428.17
5	1.0	50,485	2329.97
6	1.2	42,189	2227.73
7	1.4	34,546	2119.14
8	1.6	27,465	2001.03
9	1.8	20,899	1868.92
10	2.0	14,878	1716.72

By interpolation, when $\frac{a}{D} = 0$, $W_{ao} = 105{,}600$ Btu/hr.ft^2

$T_o = 2800^oR$

The average temperature is 2305^oR

4. Discussion of Results.

The mean temperature as obtained from the above calculation is 2305^oR (A)

In some cases the temperature of the wall is considered as the arithmetic mean of the temperatures of the source and the sink.

$$T_{mean} = \frac{3000 + 700}{2} = 1850^{\circ}R \qquad (B)$$

In Chemical Engineering books[2], the logarithmic mean is considered as the temperature of the refractory wall as an approximation. In that case,

$$T_{log\ mean\ ave.} = \frac{T_{S1} - T_{S2}}{\ln \frac{T_{S1}}{T_{S2}}}$$

$$= \frac{3000 - 700}{\ln \frac{3000}{700}} = 1580^{\circ}R \qquad (C)$$

Sometimes the refractory wall is considered at a uniform temperature T_R as considered by Rohsenowv and Choi[7]. The net radiant heat transfer through the refractory wall is zero. Therefore by heat balance,

$$q_{rnet} = A_1F_{1R}\ \sigma(T_{S1}^4 - T_R^4) + A_2F_{2R}\ \sigma(T_{S2}^4 - T_R^4) = 0$$

In this particular case $A_1 = A_2$, and also by the principle of reciprocity, $F_{1-2} = F_{2-1}$

$F_{1R} = 1-F_{1-2}$ and $F_{2R} = 1-F_{2-1}$

$$F_{1R} = F_{2R}$$

$$T_R^4 = \frac{T_{S1}^4 + T_{S2}^4}{2} = \frac{(3000)^4 + (700)^4}{2}$$

$$T_R = 2520^{\circ}R \qquad (D)$$

Thus from the above values of the temperatures, it may be concluded that the actual average temperature of the refractory wall is higher than the values obtained by the arithmetic mean and the logarithmic mean but lower than the temperature obtained on the assumption of uniform temperature of the wall.

IV. CONCLUSION

It is seen from the above calculations that the temperature of the refractory wall is varying from point to point. The average temperature is quite higher than most of the values obtained by the usual approximate methods but lower than the value obtained by the assumption of the uniform temperature of the refractory wall.

The variation in temperature of the refractory wall is nearly linear except at the end where it is slightly curved showing a sudden cooling near the sink (See Fig. 6).

It is also evident that the variation in radiant heat flux along the refractory wall is not linear (See Fig. 7), as assumed by Hottel and Keller[(3)], because the radiant heat flux varies as the fourth power of the temperature. Thus by the assumption of linear variation of the radiant heat flux along the wall, the actual quantity of radiant heat transfer will differ at higher temperature differences. More accurate results can be obtained by dividing the wall surface into a greater number of strips.

The results obtained by this method are fairly accurate but a slight discrepancy between actual radiation and calculation may be present due to some of the assumptions.

First, no surface is completely black. All surfaces reflect part of the radiation which is incident upon them. Due to these reflections, the reradiation factor may be increased. By proper selection of materials like firebricks

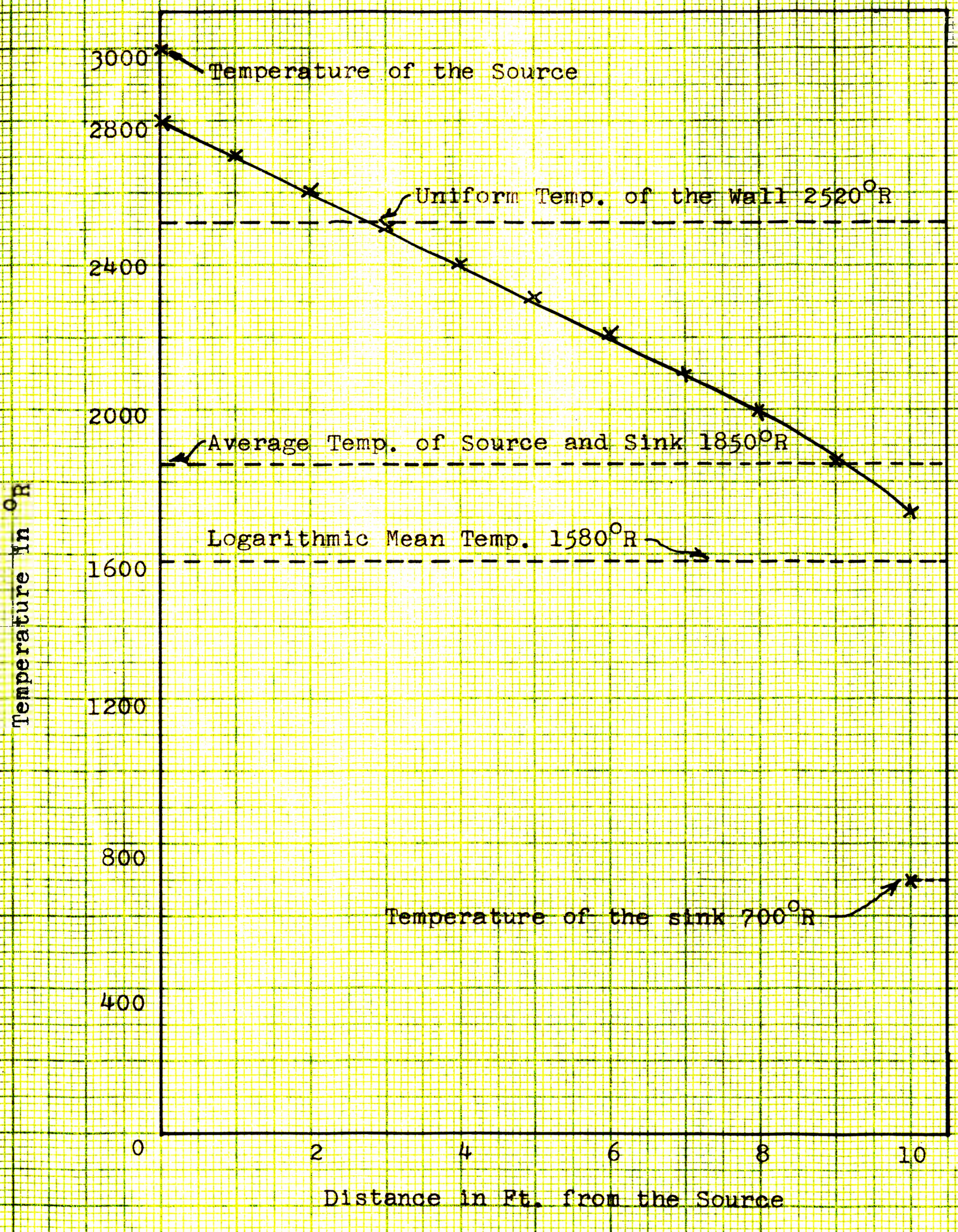

Fig. 6 Temperature Variation Along the Refractory Wall

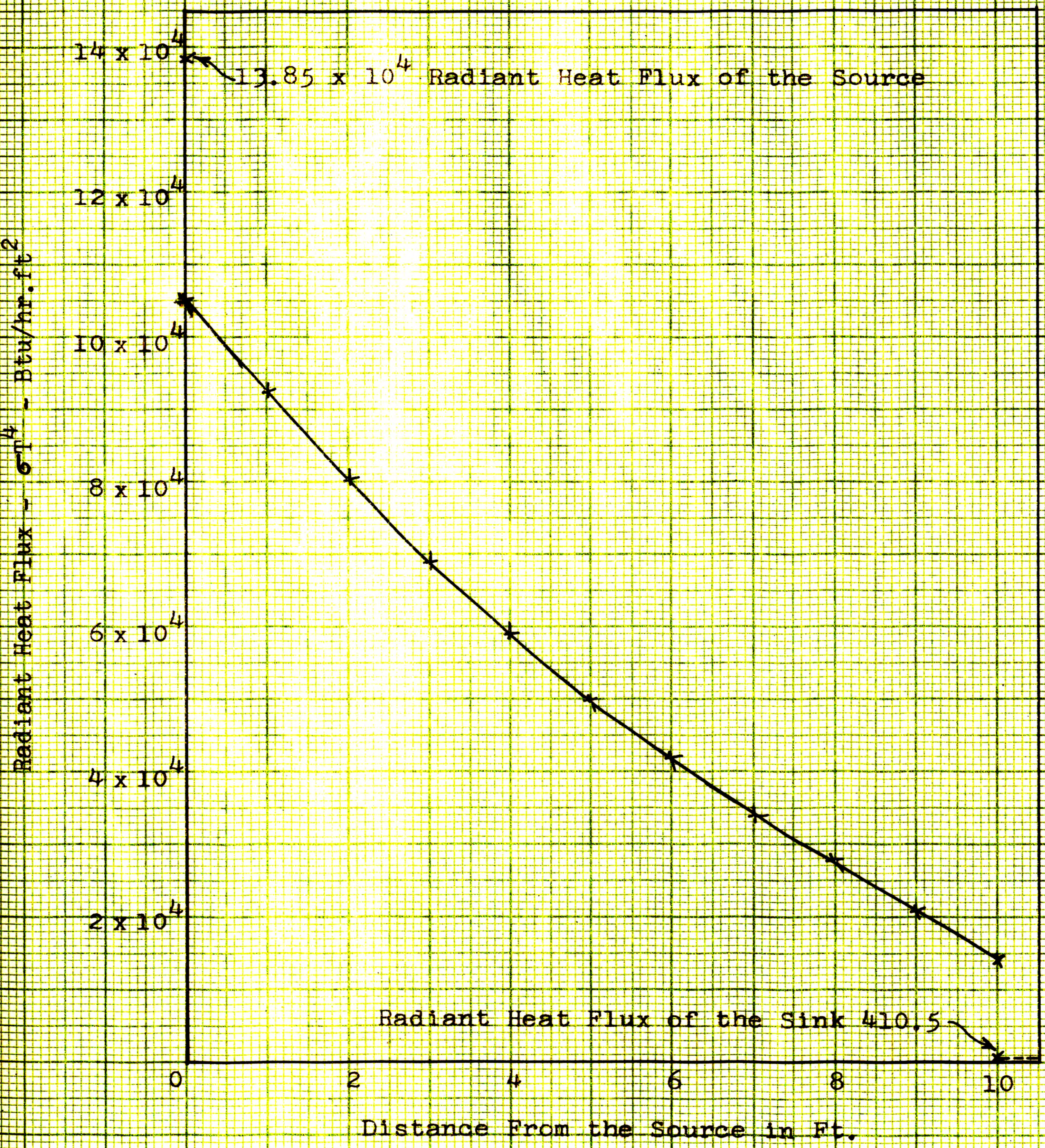

Fig. 7 Variation of Radiant Heat Flux or Emissive Power of Refractory Walls

or rough iron surfaces which absorb 90 to 95% of black body radiation this factor can be made negligible.

Second, the effect of conduction through the refractory wall may affect the reradiation by transferring heat from the inner surface to the outer surface of the walls if the thickness of the walls is considerable. But this factor is very negligible in most of the cases as the conductivity of refractory materials is very low.

In case of furnace openings, the interior of the furnace can be reduced to a disc type source and the exterior can be considered as a disc type sink. Of course, the condition of steady state may not exist if the door is opened frequently. In case of the furnaces where most of the time the door remains closed, the temperature at the opening will be almost the same as the interior of the furnace and hence the above method holds good.

Thus it may be concluded that the above method is fairly accurate for the consideration of reradiation through the refractory walls.

V. APPENDIX

Computer Program for the Solution of Equations

From Equation (18)

$$W_{a_1}\left[\int_0^{a_1} f(a_1-x)\,dx-1\right]+W_{a_2}\int_{a_1}^{a_2} f(x-a_1)\,dx+W_{a_3}\int_{a_2}^{a_3} f(x-a_1)\,dx$$
$$+W_{a_4}\int_{a_3}^{a_4} f(x-a_1)\,dx+W_{a_5}\int_{a_4}^{a_5} f(x-a_1)\,dx+W_{a_6}\int_{a_5}^{a_6} f(x-a_1)\,dx$$
$$+W_{a_7}\int_{a_6}^{a_7} f(x-a_1)\,dx+W_{a_8}\int_{a_7}^{a_8} f(x-a_1)\,dx+W_{a_9}\int_{a_8}^{a_9} f(x-a_1)\,dx$$
$$+W_{a_{10}}\int_{a_9}^{a_{10}} f(x-a_1)\,dx=-W_{b_1}\phi(a_1)-W_{b_2}\phi(2-a_1)$$

$$W_{a_1}\int_0^{a_1} f(a_2-x)\,dx+W_{a_2}\left[\int_{a_1}^{a_2} f(a_2-x)\,dx-1\right]+W_{a_3}\int_{a_2}^{a_3} f(x-a_2)\,dx$$
$$W_{a_4}\int_{a_3}^{a_4} f(x-a_2)\,dx+W_{a_5}\int_{a_4}^{a_5} f(x-a_2)\,dx+W_{a_6}\int_{a_5}^{a_6} f(x-a_2)\,dx$$
$$+W_{a_7}\int_{a_6}^{a_7} f(x-a_2)\,dx+W_{a_8}\int_{a_7}^{a_8} f(x-a_2)\,dx+W_{a_9}\int_{a_8}^{a_9} f(x-a_2)\,dx$$
$$+W_{a_{10}}\int_{a_9}^{a_{10}} f(x-a_2)\,dx=-W_{b_1}\phi(a_2)-W_{b_2}\phi(2-a_2)$$

$$W_{a_1}\int_0^{a_1} f(a_3-x)\,dx+W_{a_2}\int_{a_1}^{a_2} f(a_3-x)\,dx+W_{a_3}\left[\int_{a_2}^{a_3} f(a_3-x)\,dx-1\right]$$
$$+W_{a_4}\int_{a_3}^{a_4} f(x-a_3)\,dx+W_{a_5}\int_{a_4}^{a_5} f(x-a_3)\,dx+W_{a_6}\int_{a_5}^{a_6} f(x-a_3)\,dx$$
$$+W_{a_7}\int_{a_6}^{a_7} f(x-a_3)\,dx+W_{a_8}\int_{a_7}^{a_8} f(x-a_3)\,dx+W_{a_9}\int_{a_8}^{a_9} f(x-a_3)\,dx$$
$$+W_{a_{10}}\int_{a_9}^{a_{10}} f(x-a_3)\,dx=-W_{b_1}\phi(a_3)-W_{b_2}\phi(2-a_3)$$

Similarly other seven eqations can be derived for the values of a_4, a_5, a_6, a_7, a_8, a_9 and a_{10}.
where

$a_1 = 0.2$	$a_4 = 0.8$	$a_7 = 1.4$	
$a_2 = 0.4$	$a_5 = 1.0$	$a_8 = 1.6$	$a_{10} = 2.0$
$a_3 = 0.6$	$a_6 = 1.2$	$a_9 = 1.8$	

$$\phi(a_i) = \frac{(a_i^2 + \frac{1}{2})}{\sqrt{a_i^2+1}} - a_i$$

$$\phi(2-a_i) = \frac{[(2-a_i)^2 + \frac{1}{2}]}{\sqrt{(2-a_i)^2+1}} - (2-a_i)$$

$$W_{b_1} = \sigma T_{s_1}^4 = 138510$$

$$W_{b_2} = \sigma T_{s_2}^4 = 410.571$$

$$\int f(a_i - x)\,dx = \left[x + \frac{2(a_i-x)^2+1}{2\sqrt{(a_i-x)^2+1}} \right]$$

$$\int f(x - a_i)\,dx = \left[x - \frac{2(x-a_i)^2+1}{2\sqrt{(x-a_i)^2+1}} \right]$$

$a_i = \frac{i}{5}$ where $i = 1, 2, 3, \ldots\ldots 10.$

```
*1604
      PHI(A)   = (A*A + 0.5)/SQRTF(A*A + 1.0) - A
      DIMENSION E(10,11), Q(10,11)
    1 READ 1000, N, CI, DC, EB1, EB2
      NP1 = N + 1
      DC = 1.0/DC
      DO 100 I = 1, N
      FI = I
      A = FI * DC
      C = CI
      D = DC
      DO 50 J = 1, N
      E(I,J) = F(I,J,A,C,D)
      C = D
   50 D = D + DC
      E(I,I) = E(I,I) - 1.0
  100 E(I,N+1) = - EB1 * PHI(A) - EB2 * PHI(2.0 - A)
      DO 110 I = 1, N
      DO 110 J = 1, NP1
  110 Q(I,J) = E(I,J)
      CALL GAUJOR (E, N, N+1, 10, 11)
      DO 150 I = 1, N
  150 PUNCH 1001, I, E(I,N+1)
      DO 160 I = 1, N
      T =(E(I,NP1)/.1713E-8)**.25
  160 PUNCH 1003, I, T
      DO 200 I = 1, N
      SUM = 0.0
      DO 180 J = 1, N
  180 SUM = SUM + Q(I,J) * E(J,NP1)
      DIFF = Q(I,NP1) - SUM
  200 PUNCH 1002, Q(I,NP1), SUM, DIFF
      GO TO 1
 1000 FORMAT (8XI2,4E10.0)
 1001 FORMAT (1X2HX(,I2,4H) = , E14.7)
 1002 FORMAT (3(5XE14.7))
 1003 FORMAT (1X2HT(,I2,4H) = ,E14.7)
      END
      FUNCTION F (I,J,A,C,D)
      IF (I-J) 100, 50, 50
   50 F = D + G(A,D) - C - G(A,C)
      RETURN
  100 F = D - G(A,D) - C + G(A,C)
      RETURN
      END
      FUNCTION G(A,X)
      T = (A-X)**2
      G = (T +  .5)/SQRTF(T + 1.0)
      RETURN
      END
        10         0.          5.    138510.    410.571
```

```
X( 1) =  9.2630853E+04
X( 2) =  8.0531396E+04
X( 3) =  6.9519439E+04
X( 4) =  5.9548855E+04
X( 5) =  5.0484906E+04
X( 6) =  4.2189632E+04
X( 7) =  3.4546093E+04
X( 8) =  2.7464840E+04
X( 9) =  2.0899024E+04
X(10) =  1.4878242E+04
T( 1) =  2.7117495E+03
T( 2) =  2.6184963E+03
T( 3) =  2.5239876E+03
T( 4) =  2.4281695E+03
T( 5) =  2.3299738E+03
T( 6) =  2.2277276E+03
T( 7) =  2.1191436E+03
T( 8) =  2.0010361E+03
T( 9) =  1.8689262E+03
T(10) =  1.7167164E+03
```

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